On Applications of Dependent Types to Parameterised Digital Signal Processing Circuits

Craig Ramsay, University of Strathclyde, Scotland

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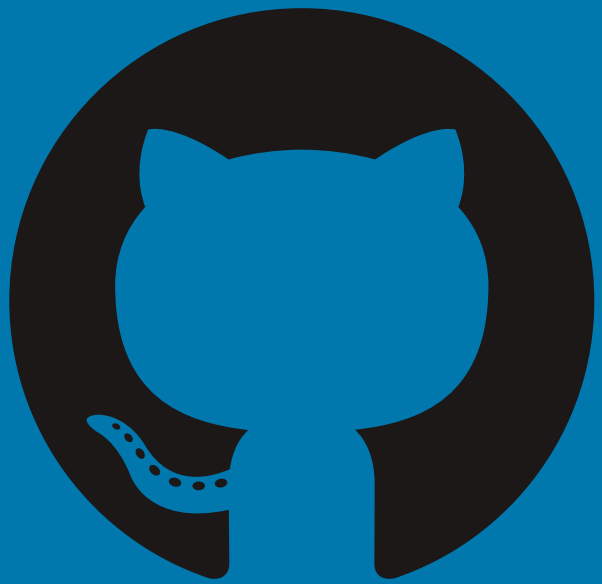

github.com/cramsay/
idris_dsp_circuits

But first, what's Type Checking?

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```
char f(int x) {
  ...
  return y;
}
```

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Let us perform computations during type checking

e.g. the ***type*** of an output can *depend* on the ***value*** of an input

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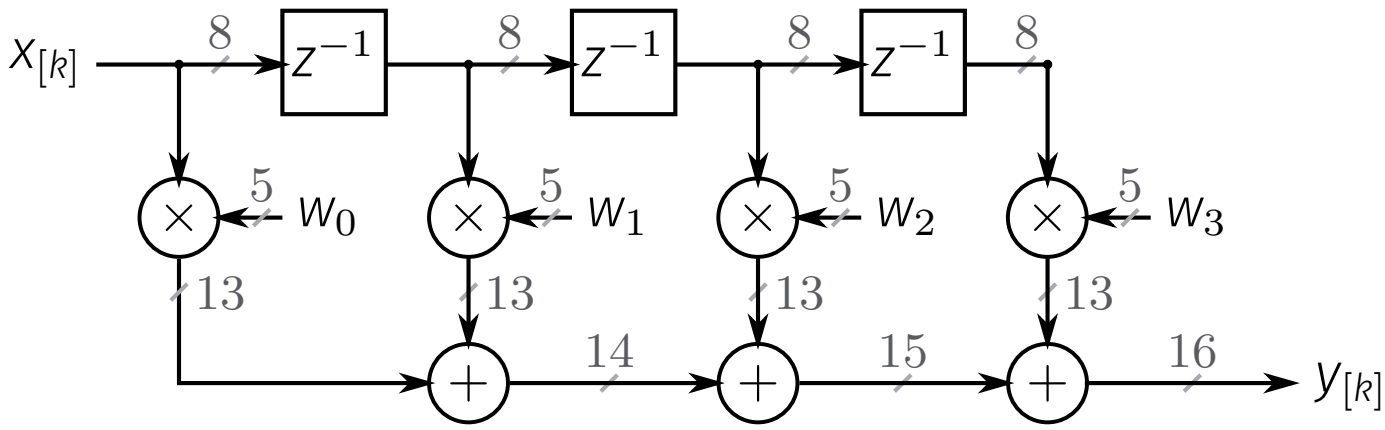
...or *prove* certain properties of a function at compile time

Applications for generating DSP circuits?

First Steps: FIR filtering...

Worst-case growth



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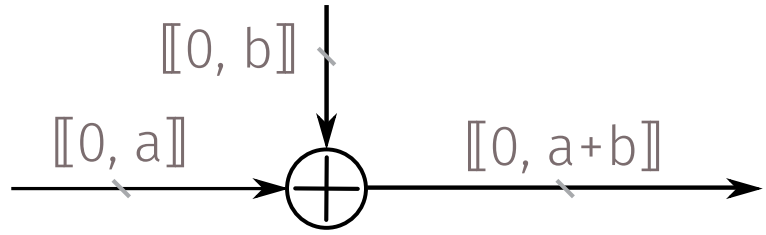


```
add : Bounded a -> Bounded b ->
      Bounded (a+b)

add (B x) (B y) = B (x+y)
```

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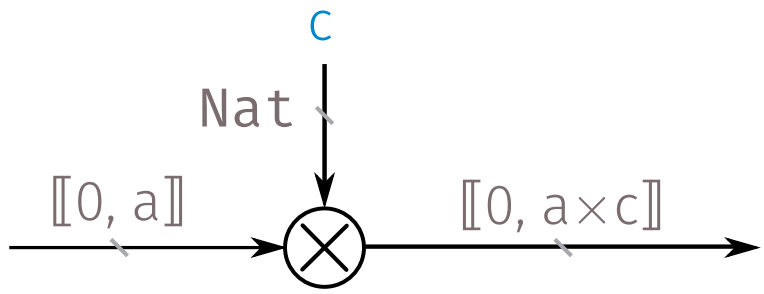


```
mulConst : Bounded a -> (c: Nat) ->
           Bounded (a*c)

mulConst (B x) c = B (x*c)
```

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dotProd : (ws : Vect j Nat)     ->
          Vect j (Bounded n)    ->
          Bounded (n * sum ws)

dotProd ws xs = ...?
```

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dotProd {j=Z}          _          _          = zeros
dotProd {j=S l} {n} (w :: ws) (x :: xs) =
  let y = add (mulConst x w) (dotProd ws xs)
  in  rewrite dotProdDistrib n w l ws in y
```

What do we gain from this?

```
fir : (ws : Vect j Nat)  ->
      Stream (Bounded n) ->
      Stream (Bounded (n * sum ws))
```

- Expressed our intent (for wordlengths) precisely

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- Expressed our intent (for wordlengths) precisely
  - A contract between `fir`'s dev and its user…
    - Mechanically checked by the type checker
  - Ensuring wordlength is theoretical minimum
  - For *all parameter sets*, not just one instance

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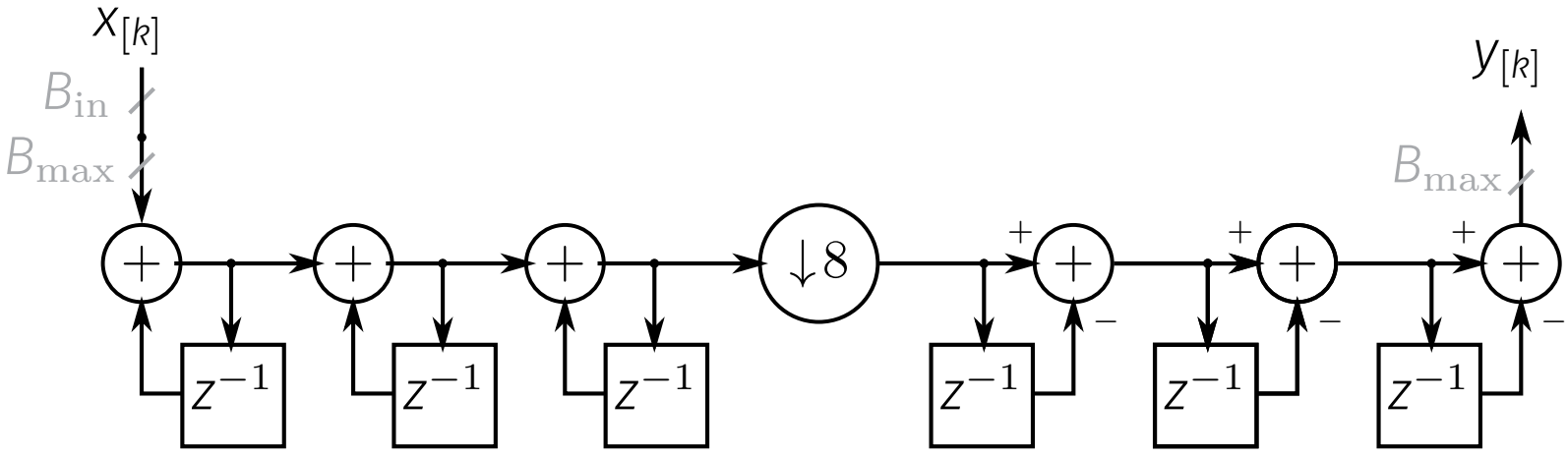
- Bad faith implementation would just resize...

We could encode arithmetic meaning with same tools, not just wordlengths.

Going Further: CIC pruning...

Without Hogenauer pruning
($R = 8$, $N = 3$ and $M = 1$)



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$$B_j = \left\lfloor -\log_2 F_j + \log_2 \sigma_{T_{2N+1}} + \frac{1}{2}\log_2 \frac{6}{N} \right\rfloor$$

where F_j is the variance error for the j^{th} stage, $\sigma_{T_{2N+1}}$ is the total variance at the output due to truncation, N is the number of stages.

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Note that the first two terms have complicated definitions of their own, including cases, sums, exponentials, and binomial coefficients.

Going Further: CIC pruning...

```
parameters (r, n, m, b_in, b_out : Nat)

  cicStage : (j : Nat) ->
             Stream (Unsigned (hogenauer  j    )) ->
             Stream (Unsigned (hogenauer (j+1)))

  cicStage = ...?
```

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- Type-level pipeline depth, arithmetic intent[1], or maybe even full z-transform
- Proving equivalence between whole circuit families...
  (Enhances peer-reviewed proofs for optimisations — the implementation is computer-checked too)
- User can do all of this — we just need to support dependent types

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- Proto-Quipper-D — Dependently typed tool for quantum circuits
  (Great tool from a different domain with similar challenges)

Thanks! Q&A?